

## Exercise 45

For what value of the constant  $c$  is the function  $f$  continuous on  $(-\infty, \infty)$ ?

$$f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \geq 2 \end{cases}$$

### Solution

$cx^2 + 2x$  and  $x^3 - cx$  are polynomials. These are continuous on their respective domains by Theorem 7. Determine  $c$  by requiring the function to be continuous at  $x = 2$ .

$$\begin{aligned} \lim_{x \rightarrow 2^-} f(x) &= \lim_{x \rightarrow 2^+} f(x) \\ \lim_{x \rightarrow 2^-} (cx^2 + 2x) &= \lim_{x \rightarrow 2^+} (x^3 - cx) \\ c(2)^2 + 2(2) &= (2)^3 - c(2) \\ 4c + 4 &= 8 - 2c \end{aligned}$$

Solve for  $c$ .

$$\begin{aligned} 4c + 2c &= 8 - 4 \\ 6c &= 4 \\ c &= \frac{2}{3} \end{aligned}$$

Below is a graph of  $f(x)$  versus  $x$  with  $c = \frac{2}{3}$ .

